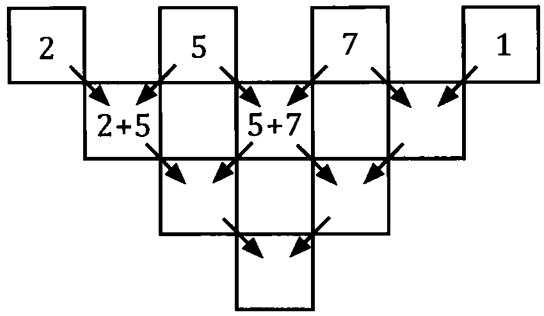
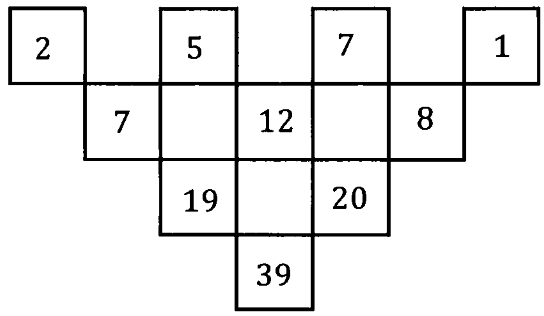
Chapter Seven.
Using equations to solve problems.

Pyramids

The pyramid pattern shown on the right is to be completed by adding the 2 and the 5 and putting the answer in the box indicated by the arrows, adding the 5 and the 7 and putting the answer in the box indicated by the arrows etc.



The completed pyramid is shown on the right.



The pyramids shown below all follow this style. Copy and complete each pyramid.

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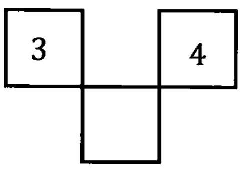

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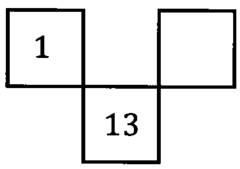

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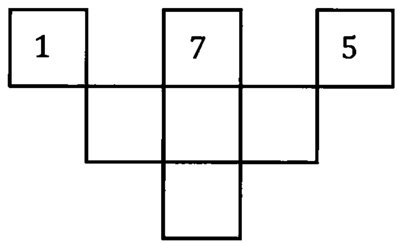

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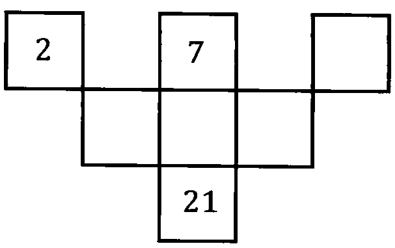

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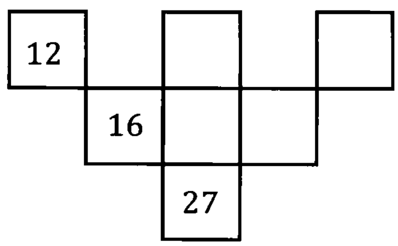

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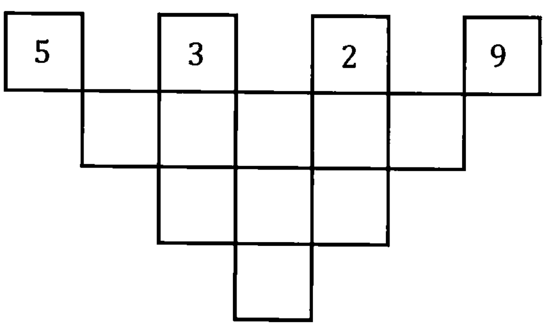

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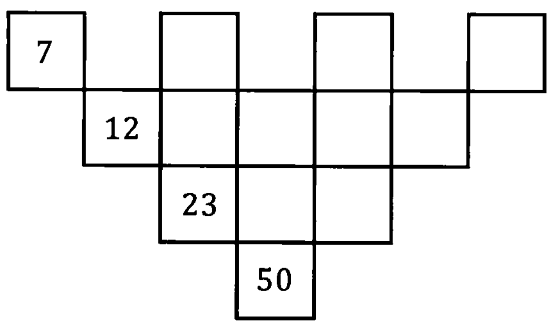
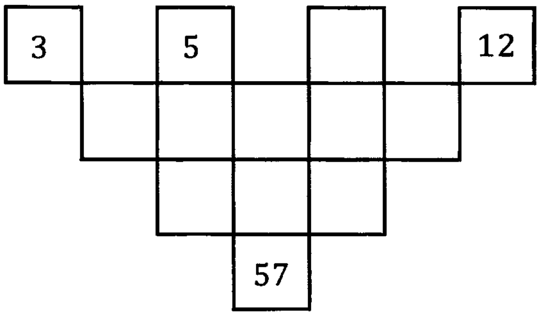


Pyramids 1 to 7 should not have caused you too much trouble but now try numbers 8 to 14 below. They are not so easy and may involve fractions.

8.



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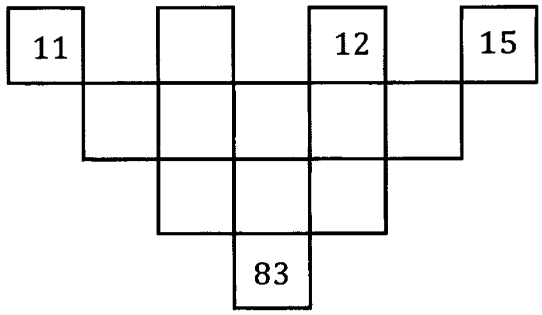

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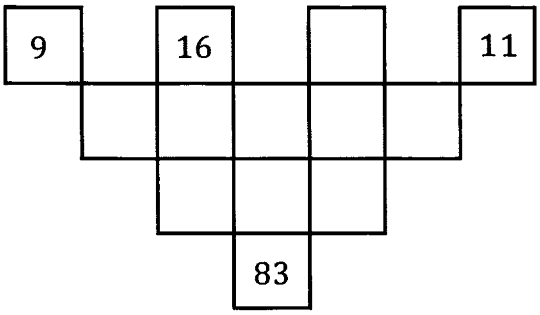

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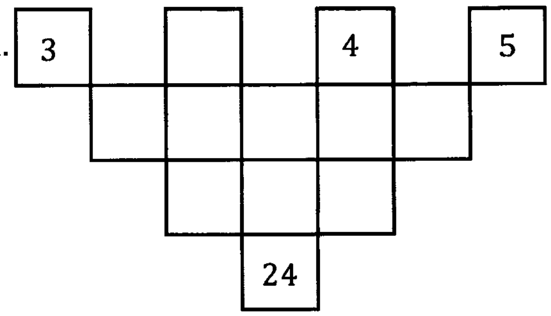

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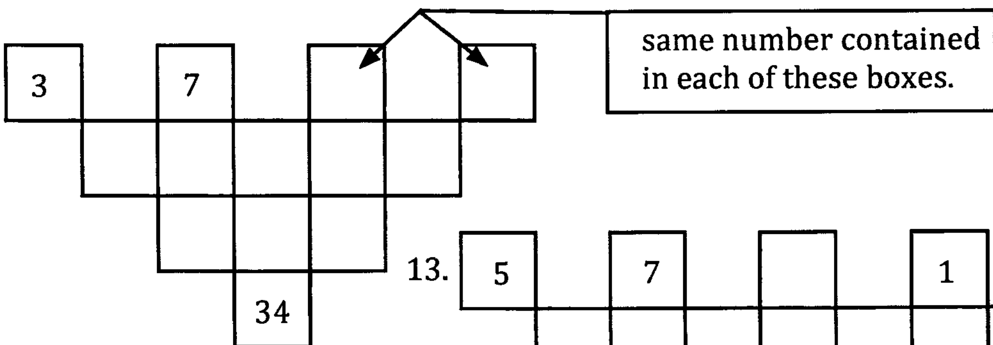

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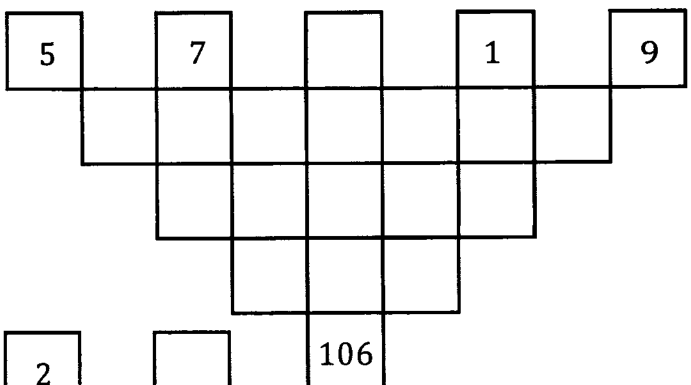

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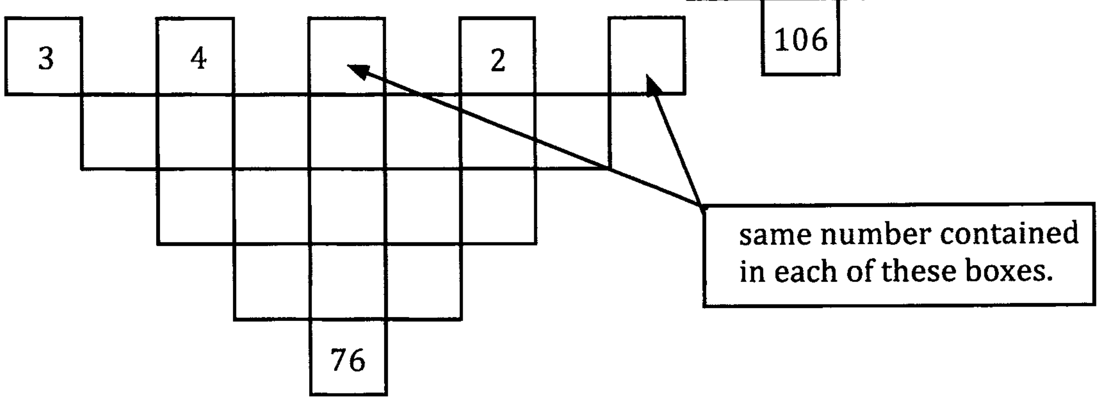
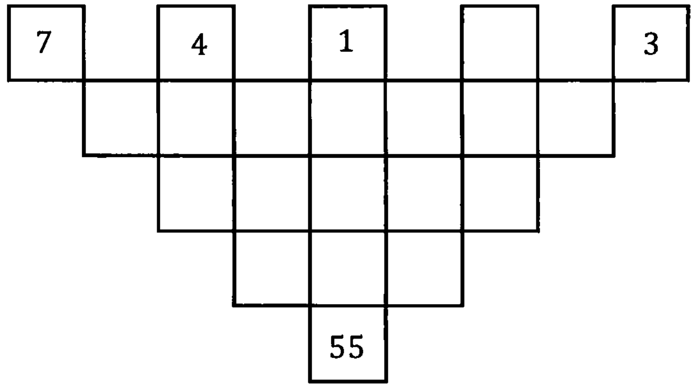


How did you get on with the pyramids on the previous pages? Did you develop any techniques for finding the missing numbers in pyramids 8 to 14?

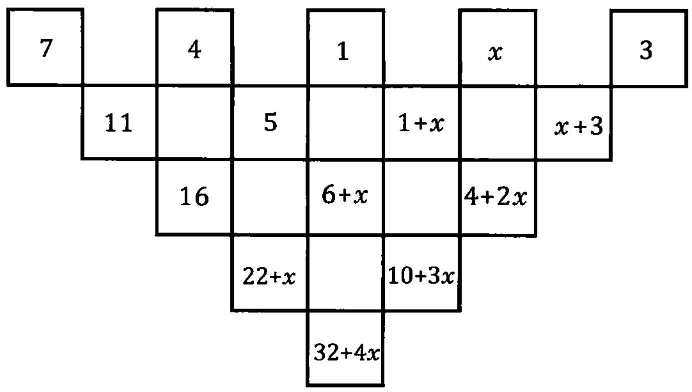
When asked to complete a pyramid like the one on the right a common initial reaction is "we don't have enough information". In fact, as you probably found out from the similar questions on the previous page, there is enough information but the empty box in the first line can make it seem that we cannot "get started".



To overcome this "getting started" difficulty we could use trial and adjustment. For example we could try some number, say 2, in the empty box in the first row and complete the pyramid using this number. If the last box is then less than 55 our initial guess needs to be increased. If the last box is greater than 55 then our initial guess needs to be decreased. Thus we "get started" by trying a number and then, based on information this number produces, we adjust and improve our initial trial. Hence the name **trial and adjustment**.

Alternatively we could overcome the "I can't get started" problem by using a symbol, for example a letter, to represent the unknown number in the top row of the pyramid.

Following through the pyramid using this symbol, say x, we obtain the entry for the last box in terms of x.



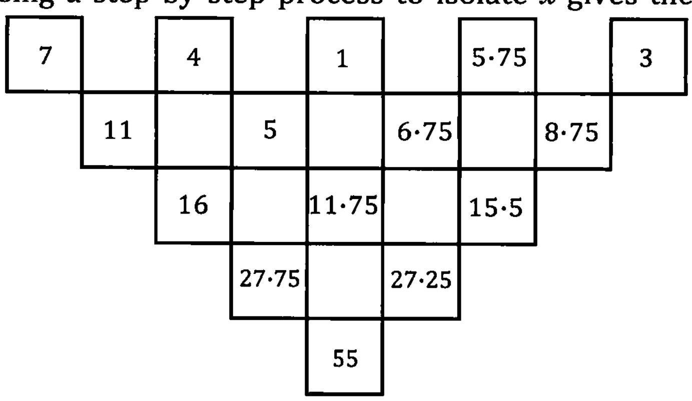
Thus, for this pyramid , the last box, $32 + 4x$, must equal 55.

Thus $\quad 32 + 4x = 55$

Solving this equation by one of the methods of the previous chapter, i.e. mentally, using the solve facility on a calculator or using a step by step process to isolate x gives the solution to this equation as

$$x = 5{\cdot}75$$

The pyramid can then be completed:



This technique of introducing a symbol, frequently x, is a useful mathematical technique for solving questions in which we seem to have sufficient information but we can't seem to "get started".

Exercise 7A

Find the value of x in each of the following pyramids.

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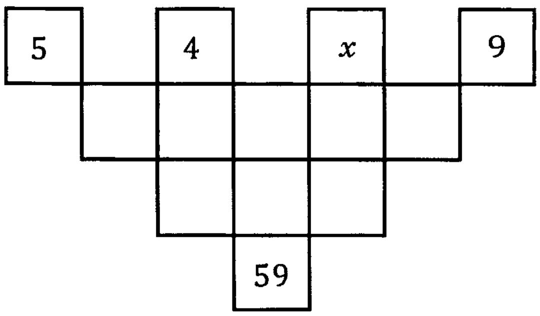

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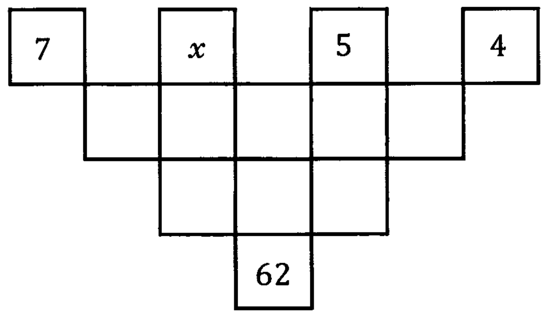

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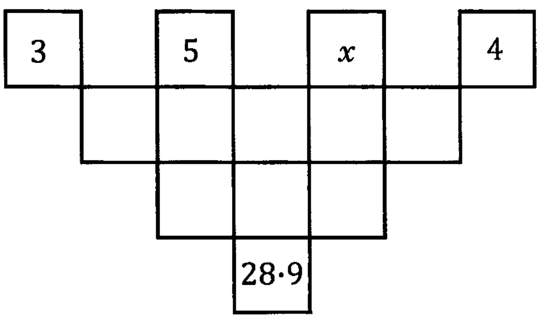

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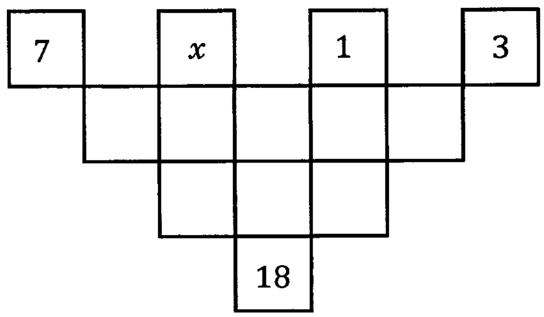

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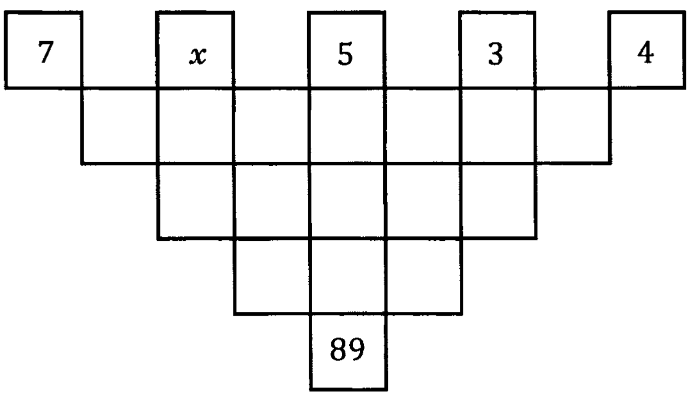

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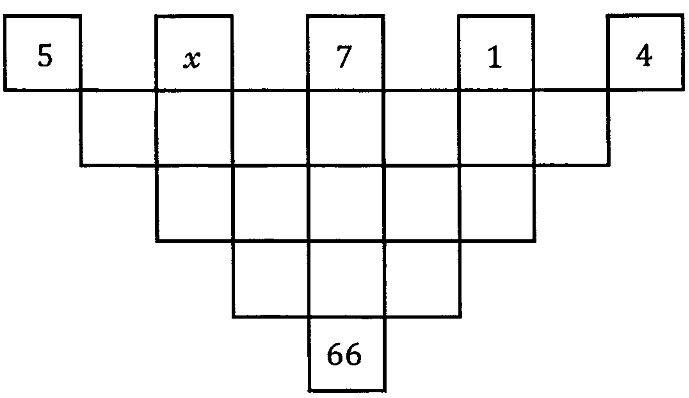
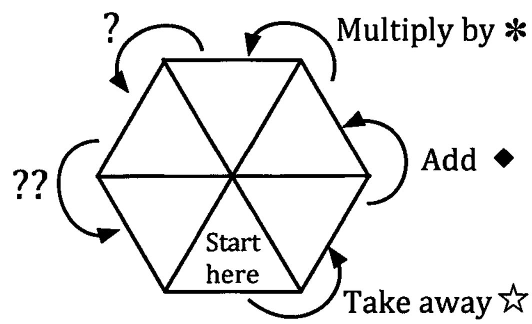
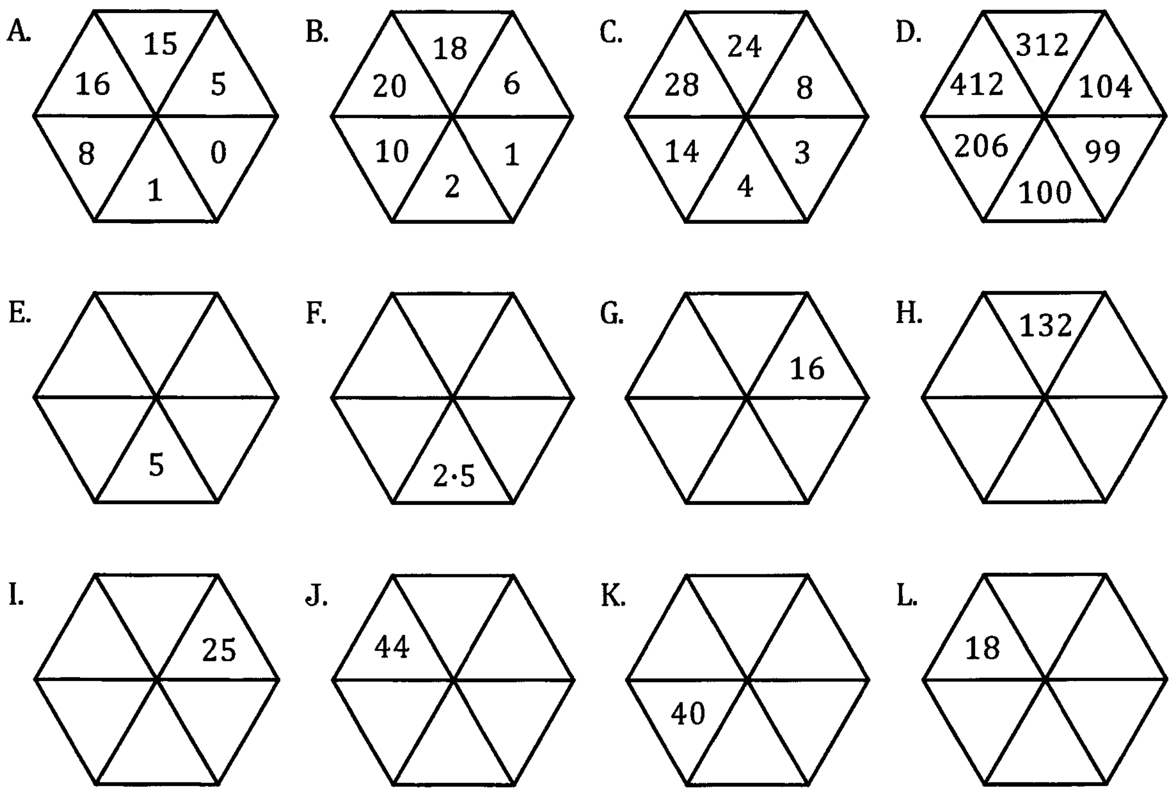


7. The "hexapatterns" shown below **all** follow the pattern shown on the right. The symbols ☆, ◆ and ∗ all represent numbers.



   (a) What number does ☆ represent?
   (b) What number does ◆ represent?
   (c) What number does ∗ represent?
   (d) What instruction should go in place of "?" ?
   (e) What instruction should go in place of "??" ?
   (f) Copy and complete the "hexapatterns" E to L shown below.



Number puzzles.

So far in this chapter we have seen an x introduced to enable us to "get started" on certain problems. Once started we then obtained an equation which allowed x to be determined. This technique of introducing an x in order to get started can be used to solve other types of puzzles and problems.

Example 1
I think of a number, multiply it by three, add seven to the answer and then add the number I first thought of. If my answer is 51 what number did I think of?

Question: We need to multiply the number by three and then add seven etc. but how can we get started if we do not know what the number is?
Answer: To allow us to get started let x be the number first thought of.

Let x be the number first thought of		x
Multiply by 3		$3x$
Add seven to the answer		$3x + 7$
Add the number first thought of		$3x + 7 + x$
The result is 51	$\therefore$	$3x + 7 + x = 51$
Collect like terms		$4x + 7 = 51$

Solving: Step by step. OR By Calculator.

$$\begin{aligned} 4x + 7 &= 51 \\ 4x &= 51 - 7 \\ 4x &= 44 \\ x &= 11 \end{aligned}$$

```
solve(4x+7=51, x)
                    {x=11}
```

The number I first thought of was 11.

Note • In the last example we did not leave the answer as $x = 11$. The initial problem asked us to find the number thought of and had no mention of x which we introduced to help solve the question. Our final sentence should clearly answer this question and not simply say "$x = 11$" unless of course the question itself had introduced, and asked us to find, x.

Example 2
I think of a number, subtract it from twelve, multiply the result by five, add the number I first thought of and end up with 24. Find the number first thought of.

Let x be the number first thought of		x
Subtract it from 12		$12 - x$
Multiply by 5		$5(12 - x)$
Add the number first thought of		$5(12 - x) + x$
The result is 24	$\therefore$	$5(12 - x) + x = 24$

Solving:

$$\begin{aligned} 5(12 - x) + x &= 24 \\ 60 - 5x + x &= 24 \\ 60 - 4x &= 24 \\ 60 &= 24 + 4x \\ 36 &= 4x \\ \therefore \quad x &= 9 \end{aligned}$$

```
solve(5(12-x)+x=24, x)
                    {x=9}
```

The number I first thought of was 9.

Note • Trial and adjustment is a perfectly acceptable alternative method for solving this type of question.

For example 2:

Try 3:	Subtract 3 from 12	9
	Multiply by 5	45
	Add number first thought of	48

Thus 3 was not correct as we want to end up with 24.

Try 5:	Subtract 5 from 12	7
	Multiply by 5	35
	Add number first thought of	40

Thus 5 was not correct but was better that 3.

Continuing in this way will eventually give us the correct answer of 9. Trial and adjustment is a very useful technique but it can be a lengthy process, particularly if the answer is not an integer. In the examples that follow the calculator approach or the step by step method will tend to be shown.

Example 3

I think of a number, add five, multiply the result by four, and end up with an answer that is 29 more than the number I first thought. Find the number first thought of.

Let x be the number first thought of.

Thus $4(x+5) = x+29$ ← Check carefully that you understand how this equation has been arrived at.

Solving:

```
solve(4(x+5)=x+29, x)
                          {x=3}
```

The number I first thought of was 3.

Example 4

I think of a number, add fifteen, divide the result by two, and end up with four more than the number I first thought. Find the number first thought of.

Let x be the number first thought of.

Thus $\frac{x+15}{2} = x+4$ ← Check carefully that you understand how this equation has been arrived at.

Solving:

Multiply by two	$x + 15$	=	$2x + 8$
Subtract x	15	=	$x + 8$
Subtract 8	7	=	x
$\therefore$	x	=	7

The number I first thought of was 7.

Exercise 7B

1. If x represents "the number" write each of the following statements in terms of x.
   Example: Three times the number then add one. Answer: $3x + 1$

   (a) Multiply the number by five then add six.
   (b) Take the number away from fourteen.
   (c) Add six to the number and then multiply by five.
   (d) Double the number then take away seven.
   (e) Take seven from the number and then double your answer.
   (f) Double the number, add five and then multiply your answer by three.

2. If x is used to represent "the number I think of" in each of the statements A to H below, select the equation from the box on the right that matches each statement and then solve the equation.

   A: I think of a number, double it and add one and my answer is 10.
   B: If I take one from the number I think of and double the result my answer is ten.
   C: I think of a number and subtract it from ten and my answer is one.
   D: I think of a number, add one and then double the result and my answer is ten.
   E: I think of a number, divide it by two and then subtract one and end up with 10.
   F: I think of a number, take ten away and my answer is one.
   G: I think of a number, subtract one and then divide by two and my answer is ten.
   H: Twice the number I thought of exceeds ten by one.

$2x + 1 = 10$

$2x - 10 = 1$

$x - 10 = 1$

$1 - \frac{x}{2} = 10$

$10 - x = 1$

$\frac{x}{2} - 1 = 10$

$2(x - 1) = 10$

$\frac{x - 1}{2} = 10$

$\frac{1 - x}{2} = 10$

$2(x + 1) = 10$

3. I think of a number, multiply by three, subtract eleven from the answer and then add the number I first thought of. If my answer is 25 what was the number first thought of?

4. I think of a number, add seven, multiply the result by two and end up with an answer that is 17 more than the number I first thought of. Find the number first thought of.

5. I think of a number, double it, add seven and end up with an answer that is 17 more than the number I first thought of. Find the number first thought of.

6. I think of a number, subtract four, multiply the result by three and end up with an answer that is two less than the number I first thought of. Find the number first thought of.

7. I think of a number, subtract five, divide the answer by two and end up with seven less than the number I first thought of. Find the number first thought of.

8. I think of a number, add two, multiply the answer by three and then subtract eleven. This gives an answer equal to twice the number first thought of. Find the number first thought of.

9. I think of a number, double it, add three, multiply the result by two, take away the number I first thought of and end up with twenty seven. Find the number first thought of.

10. I think of a number, multiply it by three and then subtract one. If one quarter of this answer is added to the number I first thought of the answer is 22·5. Find the number I first thought of.

11. I think of a number, subtract it from fourteen, multiply the result by three and find that my answer exceeds the number I first thought of by four. Find the number I first thought of.

12. I think of a number, double it and add three. I find that if I take twenty one from my answer I end up with half of the number I first thought of. Find the number I first thought of.

Solving problems.

All of the questions of the previous exercise were of the "think of a number" style. They could all be solved by introducing an x to represent the number. This allowed us to "get started", form an equation and determine x. The following examples show this same technique used to solve a variety of problems.

Example 5

Jackie has saved \$28 more than John. Between them they have saved \$154. How much has each person saved?

Let the amount John has saved be	$\$x$		
The amount Jackie has saved will then be	$\$(x+28)$		
Between them they have saved \$154. Thus	$x+(x+28)$	=	154
i.e.	$2x+28$	=	154
This equation can be solved to give	x	=	63

John has saved \$63 and Jackie has saved \$91.

Note: It is not the intention here to claim that the introduction of x is the only way to solve the problem. As has been mentioned before, "trial and adjustment" can be used. Alternatively the answer can be "reasoned through". For example, the previous question could be solved as follows:

If we take Jackie's extra \$28 from the \$154 then the answer, \$126, is their total if they both had John's amount. Thus John's amount must be \$126 ÷ 2 = \$63. Hence Jackie must have saved \$63 + \$28 = \$91.

The arithmetic involved, i.e. taking 28 from 154 and then dividing by two, is the same as we would do to solve the equation $2x + 28 = 154$ by the step by step process, but the answer was "reasoned through" rather than obtaining and solving an equation in x. If you choose this "reasoning" method be sure to explain what you are doing so that others can follow your reasoning.

Example 6

Rosalyn is 8 years older than Jennifer. In six years' time their ages will be such that Rosalyn will be twice as old as Jennifer. How old is Jennifer now?

Let Jennifer's age now be	x	years
Thus Rosalyn's age now is	$(x+8)$	years
In 6 years' time Jennifer will be	$(x+6)$	years
In 6 years' time Rosalyn will be	$(x+8)+6$	years
Thus	$(x+8)+6$	$= 2(x+6)$
This equation can be solved to give	x	$= 2$

Jennifer is 2 years old now.

Note: Alternatively "trial and adjustment" can be used or the answer can be "reasoned through" as follows:

If Rosalyn is 8 years older than Jennifer now, she will always be 8 years older. Thus, in 6 years' time, when Rosalyn is twice as old as Jennifer, the difference in their ages will be both 8 years and one lot of Jennifer's age. Thus in 6 years' time Jennifer will be 8 and Rosalyn will be 16. Thus Jennifer is 2 years old now.

Example 7

An amateur drama group hire a theatre for their production. They expect to sell all of the 1200 tickets, some at \$10 and the rest at \$7. The group require the ticket sales to cover their \$4150 production costs, to allow a donation of \$4000 to be made to charity and to provide a profit of \$1000 to aid future productions. If they are to exactly achieve this target and their expectations regarding ticket sales are correct how many of the total 1200 tickets should they charge \$10 for and how many should they charge \$7 for?

Let the number of \$7 tickets be	x	
These will give an income of	$7x$	dollars
The number of \$10 tickets will then be	$(1200-x)$	
These will give an income of	$10(1200-x)$	dollars
Thus	$7x+10(1200-x)$	$= 4150+4000+1000$
which can be simplified to	$12000-3x$	$= 9150$
Solving gives	x	$= 950$

The group should sell 950 tickets at \$7 each and 250 tickets at \$10 each.

Exercise 7C

1. Tony and Bob each put some money towards the purchase of a new car they need for their business. Tony puts in \$5 500 more than Bob. Together they put in a total of \$18 500. How much does each contribute?

2. Three people, Sue, Lyn and Paul run a part time business. At the end of their first year they decide that the profits should be shared out such that Lyn gets one and a half times as much as Sue, and Paul gets \$5 000 more than Sue. If the profits for the year are \$47 000 how much should each receive?

3. Bill is 29 years older than his daughter Rebecca. In fifteen years' time their ages will be such that Bill will be twice as old as Rebecca. How old is Bill now?

4. A manufacturer sells a particular product for \$40 per unit. The manufacturer's costs for producing the units consist of a fixed \$5 000 plus a cost of \$22 per unit. If the manufacturer produces and sells x units find,
   (a) an expression in terms of x for the cost to the manufacturer for producing these x items,
   (b) the value of x for the manufacturer to at least "break even". (Assume that x must take integer values.)

5. A firm manufactures two types of chair, the deluxe and the standard. In one week the firm manufactures a total of 120 of these chairs, x of the standard and $(120 - x)$ of the deluxe. Each standard chair requires 3 hours of work and each deluxe requires 4 hours of work.
   (a) Find an expression in terms of x for the number of hours required to make the x standard chairs.
   (b) Find an expression in terms of x for the number of hours required to make the $(120 - x)$ deluxe chairs.
   (c) If the 120 chairs required 405 hours of work altogether find how many of each type of chair were made.

6. A farmer wishes to fence off a rectangular area with the length 10 metres longer than the width. If the farmer has 360 metres of fencing available for this task what should be (a) the width of the rectangle,
   (b) the length of the rectangle,
   (c) the area of the rectangle.

7. If Heidi's current age in years is multiplied by five and two is subtracted from the answer the result is equal to her Dad's age in years. If Heidi is currently x years old find an expression for her Dad's age.
   In 8 years' time the ages will be such that Heidi's Dad will be three times as old as Heidi. How old is Heidi now?

8. An amateur drama group hire a theatre for their production. They expect to sell all 850 tickets, some at \$12 and the rest at \$8. The group require the ticket sales to cover their \$3760 production costs and to make a profit of \$4000. If they are to exactly achieve this target and their expectations regarding ticket sales are correct how many of the 850 tickets should they charge \$12 for and how many should they charge \$8 for?

9. A farmer has a certain number of acres that she wishes to use to grow wheat, barley and lupins.
Whatever acreage she decides to use for the lupins she likes to have 2 000 acres more than this for barley.
She also likes to use twice as many acres for wheat as she does for barley.
If she uses x acres for lupins find (a) an expression in terms of x for the number of acres she uses for barley,
(b) an expression in terms of x for the number of acres she uses for wheat.
(c) If she decides to use a total of 18 000 acres for the three products determine how many acres she uses for each.

10. A firm making fertiliser produces a new fertiliser *QuickGrow*. Each 50 kg bag of *QuickGrow* contains x kg of compound X and $(50 - x)$ kg of compound Y. Each kilogram of X contains 150 g of a particular element and each kilogram of Y contains 80 g of this element. The company wants each 50 kg of *Quickgrow* to contain 6·24 kg of this element. Find
(a) an expression in terms of x for the amount of the particular element contained in x kg of compound X and state the units,
(b) an expression in terms of x for the amount of the particular element contained in $(50 - x)$ kg of compound Y and state the units.
(c) How much of each compound, X and Y, should each 50 kg of *Quickgrow* contain to give the desired total amount of the particular element?

11. A book shop owner orders some hard-back and some soft-back versions of a book. The hard-back version costs the shop owner \$20 each and the soft-back \$12 each. The total order was for 300 books.
When the order arrives there were only 200 books! The shopkeeper wishes to query the order but cannot find his copy of the original order. However his records do tell him that it was going to cost him \$5 080. How many of each type of the book was his original order for?

12. An investor has \$5 000 to invest and decides to invest some of it with company A and the rest with company B. After one year each \$1 invested with company A has grown to \$1·20, each dollar invested with company B has grown to \$1·05 and the \$5 000 has grown to \$5 670. How much of the original \$5 000 was invested with each company?

Equations from simple interest formula.

In Unit One of this Mathematics course you would have determined the *Simple Interest*, \$I, earned when \$P is invested for T years in an account paying R% per annum simple interest.

The formula used in this situation is: $I = \dfrac{PRT}{100}$

If instead we use R in decimal form we use $I = PRT$. For example if the interest rate is 6% the first formula would use $R = 6$ but the second would use $R = 0{\cdot}06$.

In Unit One we were determining I given P, R and T. If instead we want to determine P, or R or T, given I and the other two, we can substitute values into the appropriate formula and solve the resulting equation.

Example 8

How much money needs to be invested for 3 years at 6% simple interest to earn interest of \$864.

Using $I = PRT$

Given $I = 864$, $R = 0{\cdot}06$, $T = 3$ then

$$\begin{aligned} 864 &= P \times 0{\cdot}06 \times 3 \\ \text{i.e.} \quad 864 &= 0{\cdot}18P \\ \text{and so} \quad P &= \frac{864}{0{\cdot}18} \\ &= 4800 \end{aligned}$$

The investment needs to be \$4800.

Example 9

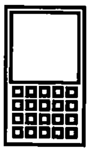
How long does an investment of \$12 500 need to be invested at 3·8% per annum simple interest to earn interest of \$2 375?

Using $I = PRT$

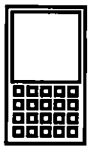
Given $P = 12500$, $R = 0{\cdot}038$, $I = 2375$

$$\begin{aligned} \text{then} \quad 2375 &= 12500 \times 0{\cdot}038 \times T \\ \text{i.e.} \quad 2375 &= 475T \\ \text{and so} \quad T &= \frac{2375}{475} \\ &= 5 \end{aligned}$$

The investment needs to be for 5 years.



Alternatively questions like the previous two examples could be solved using the built in capability of some calculators to perform simple interest calculations.



Equations from ratios.

The *Preliminary work* at the beginning of this text reminded us of the idea of a ratio. In particular the following example was given:

> Suppose the ratio of males to females in a school is 17 : 21.
> If we know that there are 231 females in the school we can determine the number of males
>
> $$\begin{array}{rcl} \text{males : females} & = & 17 : 21 \\ & = & ? : 231 \end{array} \quad \times 11$$
>
> 231 / 21
> 11
>
> The number of males = 17 × 11, i.e. 187.

Alternatively. if we let the number of males be m we could set up, and solve, an equation, as follows:

$$\begin{array}{llrcl} & & \text{males : females} & = & 17 : 21 \\ \therefore & & m : 231 & = & 17 : 21 \\ \text{Hence} & & \dfrac{m}{231} & = & \dfrac{17}{21} \\ \times \text{ by 231 to eliminate fractions:} & & 231 \times \dfrac{m}{231} & = & 231 \times \dfrac{17}{21} \\ & & m & = & 231 \times \dfrac{17}{21} \\ & & & = & 187 \end{array}$$

The number of males is 187, as before.

Suppose the previous situation had instead given us the ratio as:

$$\text{females : males} = 21 : 17.$$

We would then have proceeded as follows:

$$\begin{array}{lrcl} \therefore & 231 : m & = & 21 : 17 \\ \text{Hence} & \dfrac{231}{m} & = & \dfrac{21}{17} \end{array}$$

Now with the variable in the denominator this is not a linear equation but this need not trouble us. We could solve the equation by:

- ☞ first multiplying by $17m$ to eliminate the fractions, as shown below left,

or ☞ using the fact that if $\frac{a}{b} = \frac{c}{d}$ then it follows that $\frac{b}{a} = \frac{d}{c}$, as shown below right.

$$\begin{array}{lrcl} \text{Given} & 231 : m & = & 21 : 17 \\ & \dfrac{231}{m} & = & \dfrac{21}{17} \\ \times \text{ by } 17m & 17m \times \dfrac{231}{m} & = & \dfrac{21}{17} \times 17m \\ \text{Hence} & 17 \times 231 & = & 21m \\ \text{Solving gives} & m & = & 187 \end{array}$$

$$\begin{array}{lrcl} \text{Given} & 231 : m & = & 21 : 17 \\ & \dfrac{231}{m} & = & \dfrac{21}{17} \\ \text{Hence} & \dfrac{m}{231} & = & \dfrac{17}{21} \\ \times \text{ by 231} & 231 \times \dfrac{m}{231} & = & \dfrac{17}{21} \times 231 \\ \text{Hence} & m & = & 187 \end{array}$$

Notice that if $\qquad \dfrac{a}{b} = \dfrac{c}{d}$

Then multiplying by bd: $\qquad bd \times \dfrac{a}{b} = bd \times \dfrac{c}{d}$

gives $\qquad ad = bc$

Notice that this final statement could have been obtained by "cross multiplying" the original "fraction equals fraction" equation. *Cross multiplying* is a useful short cut but it needs to be used with care and with an understanding of why it works. Only use it in situations that are "single fraction = single fraction".

$\dfrac{a}{b} \quad \dfrac{c}{d}$

Example 10

Find the values of a and b in the following

(a) $a : 4 = 7 : 20$

(b) $15 : 4b = 1 : 2$

(a) Given $\quad a : 4 = 7 : 20$

then $\quad \dfrac{a}{4} = \dfrac{7}{20}$

Hence $\quad 20 \times a = 7 \times 4$

$a = \dfrac{28}{20}$

$= 1{\cdot}4$

(b) Given $\quad 15 : 4b = 1 : 2$

then $\quad \dfrac{15}{4b} = \dfrac{1}{2}$

Hence $\quad 15 \times 2 = 4b \times 1$

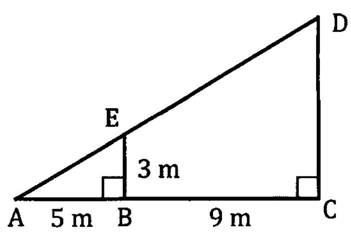
$b = \dfrac{30}{4}$

$= 7{\cdot}5$

In your study of unit one of this course you would have encountered the idea of **similar triangles**. Situations involving determining unknown lengths in similar triangles can often involve setting up then and solving a statement involving ratios, as the next two examples show.

Example 11

In the diagram shown on the right AB = 5 m, BC = 9 m, and BE = 3 m. Find the length of CD, justifying your answer.



In triangles ABE and ACD: $\quad \angle EAB = \angle DAC$ (same angle)

$\angle EBA = \angle DCA \quad (= 90°)$

Hence the third angles will be equal and so $\Delta ABE \sim \Delta ACD$, corresponding angles equal.

Hence $\quad AB : AC = BE : CD$

Letting the length of CD be x m: $\quad 5 : 14 = 3 : x$

i.e. $\quad \dfrac{5}{14} = \dfrac{3}{x}$

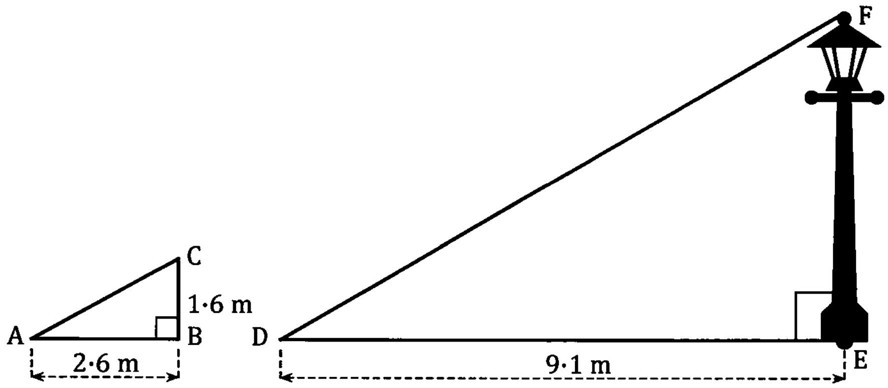
Hence $\quad 5x = 42$

and so $\quad x = 8{\cdot}4$

CD is of length 8·4 m.

Example 12. So how tall is the street light?

On a sunny day, at the same time that a street light standing on horizontal ground casts a shadow of length 9·1 metres, a 1·6 metre stick held vertically on the same ground casts a shadow of length 2·6 metres.



What does this information suggest that the height of the street light is?

In triangles ABC and DEF: $\angle CAB = \angle FDE$ (angle sun's rays make with the ground)
$\angle CBA = \angle FED$ $(= 90°)$

Hence the third angles will be equal and so $\Delta ABC \sim \Delta DEF$, corresponding angles equal.

Hence $\quad AB : DE = BC : EF$

Let the height of the street light be h metres.

$$2{\cdot}6 : 9{\cdot}1 = 1{\cdot}6 : h$$

i.e. $$\frac{2{\cdot}6}{9{\cdot}1} = \frac{1{\cdot}6}{h}$$

Hence $\quad 2{\cdot}6h = 9{\cdot}1 \times 1{\cdot}6$

and so $\quad h = 5{\cdot}6$

The information suggests that the height of the street light is 5·6 metres.

Exercise 7D

1. Find the values of $a, b, c, \ldots i$ in the following.

(a) $a : 9 = 2 : 3$	(b) $b : 10 = 2 : 5$	(c) $4 : 3 = 8 : c$
(d) $d : 3 = 5 : 2$	(e) $2e : 9 = 4 : 5$	(f) $f : 2 = 7 : 5$
(g) $6 : g = 4 : 1$	(h) $17 : 2h = 5 : 1$	(i) $5 : 8 = 3 : i$

2. Determine how much money needs to be invested for 3 years at 8% per annum simple interest to accrue interest of $1008.

3. What annual rate of simple interest would cause an investment of $6400 to grow to $7360 in two years?

4. Sally invests a sum of money into an account paying 8·2% simple interest. After three years the account has earned interest of \$209·10. What was the initial sum invested?

5. For how long must an investment of \$10 000 be invested in an account paying 8·6% per annum simple interest for it to become \$13 870?

6. How many days must a sum of \$8650 be left to accrue interest at a rate of 5% per annum simple interest to become \$8823?

7. What annual rate of simple interest is needed to see an initial investment of \$6720 become \$7011·20 in 8 months.

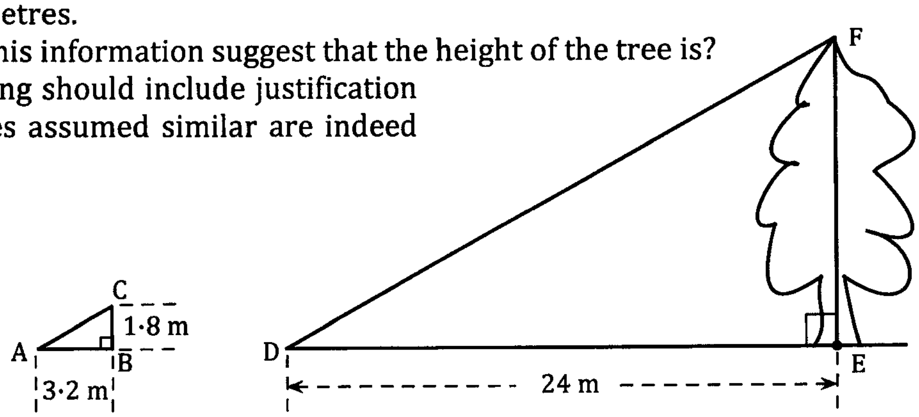
8. \$65000 is invested in an account paying simple interest at a rate of R% per annum. Nine months later the account is closed and the total balance of principal plus interest is then \$68997·50. Find R.

9. What annual rate of simple interest is required to see an investment of eight and a half thousand dollars grow to eight thousand eight hundred and six dollars in 219 days?

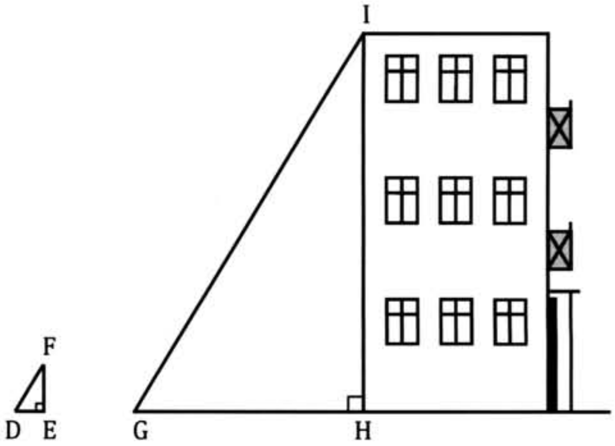
10. The ratio of males to females in the audience for a particular event was 5 : 7. If there were 1045 males in the audience how many females were in the audience?

11. Suppose the ratio of male students to female students in a school is 15 : 17. If there are 345 male students in the school how many female students are there in the school?

12. **So how tall is the tree?**
At the same time as a tree, stood on horizontal ground, casts a shadow of length 24 metres, a 1·8 metre stick held vertically on the same ground casts a shadow of length 3·2 metres.
What does this information suggest that the height of the tree is?
(Your working should include justification that triangles assumed similar are indeed similar.)

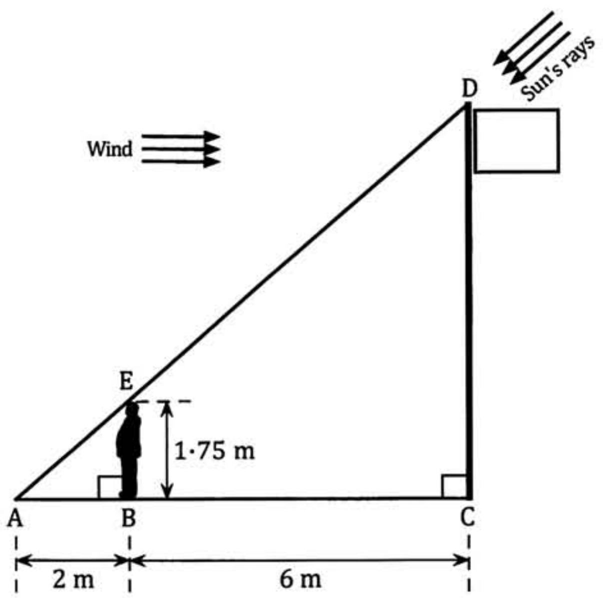


13. **So how tall is the building?**
When a vertical sign post that is known to have a height of 3 metres casts a 1·8 metre shadow a nearby building casts a shadow of length 15 metres.
How tall is the building?
(Your working should include justification that triangles assumed similar are indeed similar.)

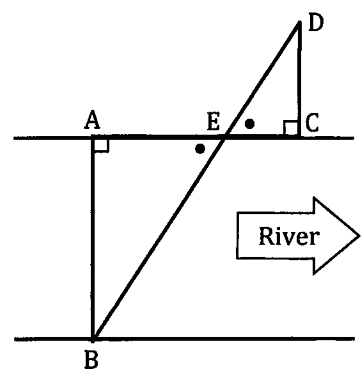


14. **So how tall is the flag pole? I.**
A stick of length 2·1 metres held vertically on horizontal ground casts a shadow of length 0·6 metres.
At the same time, a nearby flagpole casts a shadow of length x metres.
If the height of the flagpole is h metres find an expression for h in terms of x.
If $x = 1{\cdot}5$ find h.
(Your working should include justification that triangles assumed similar are indeed similar.)

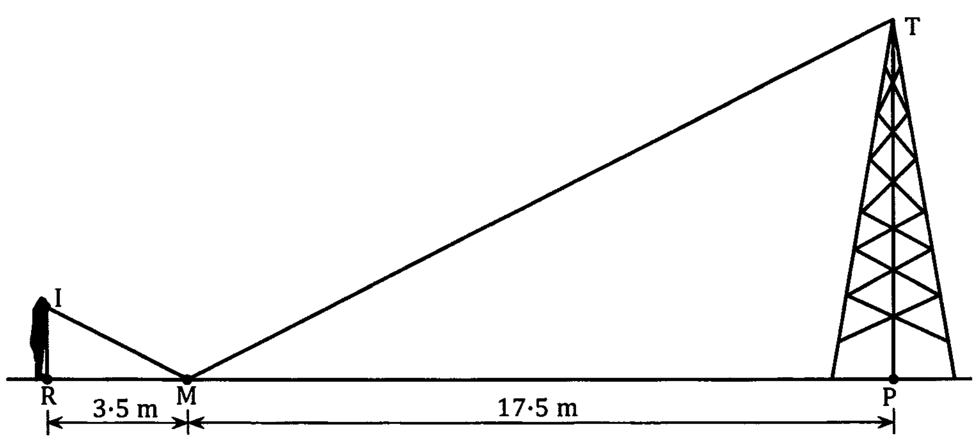
15. **So how tall is the flag pole? II.**
Standing on level ground, and with the sun shining, Panji starts at the base of a flagpole and walks along the flagpole's shadow until the tip of his own shadow is at the same point on the ground as the tip of the shadow cast by the flagpole.
Panji, who's own height is 1·75 metres, is then 2 metres from the tip of his shadow and 6 metres from the base of the flagpole.
How tall is the flagpole?
(Your working should include justification that triangles assumed similar are indeed similar.)



16. **So how wide is the river?**
As part of an initiative test a team of trainee soldiers is set the task of estimating the width of a river, from one side of the river. To do this they note a tree at waters edge on the far side (point B in the diagram on the right) and put a pole at point A on "their" river bank, directly opposite B. They then locate some suitable point C on the bank on their side and a point D as shown. Standing at D and looking at the tree on the opposite bank allows them to locate, and mark, point E.
They then measure AE as being 34·6 metres, EC as 19·2 metres and CD as 29·8 metres.
How wide is the river?
(Your working should include justification that triangles assumed similar are indeed similar.)



17. **So how tall is the pylon?**
Roz places a small flat mirror on level ground between herself and an electricity pylon. Roz notices that with herself, the mirror and the pylon in line she can see the top of the pylon in the mirror when she is 3·5 metres from the mirror and the mirror is 17·5 metres from the middle of the base of the pylon.
If Roz's eye height is 172 centimetres how tall is the pylon?
(Your working should include justification that triangles assumed similar are indeed similar.)



18. Mary takes out a loan which involves simple interest charged at the rate of 8% per annum. After 18 months Mary repays $1344 which clears the loan and interest.
How much did Mary borrow in the first place?

19. Mai takes out a loan which involves simple interest charged at the rate of 5% per annum. After 219 days Mai repays $7004 which clears the loan and interest.
How much did Mai borrow in the first place?

THINK OF A NUMBER

Think of a whole number between one and ten (and remember it).

Double it

Add 3

Double your answer

Add on the number you first thought of

Add four

Divide by 5

Add 1

Take away the number you first thought of.

Your answer is 3 (or at least it should be if you have followed the instructions correctly!)

Why does the above puzzle always work no matter what whole number between one and ten is chosen as the starting number? Write an explanation of why the puzzle works.

Does it work for numbers other than whole numbers between one and ten? Explain.

A TRICK INVOLVING FIFTEEN COINS

Ranii placed fifteen coins on the table and said to her sister Jenna,

"While I shut my eyes you put some of the coins in your left hand, the rest in your right hand, and then put your hands behind your back so that I cannot see them."

Jenna followed the instructions and then Ranii said,

"Now multiply the number of coins you have in your left hand by two, add on the number of coins you have in your right hand and tell me the answer."

Again Jenna followed the instructions and announced,

"That makes 22."

Performing some simple arithmetic Ranni was quickly able to announce,

"You have 7 in your left hand and 8 in your right."

"That's right," exclaimed Jenna, *"How did you know that?"*

How did she know it? **Explain what she did and why it works.**

Miscellaneous Exercise Seven.

This miscellaneous exercise may include questions involving the work of this chapter, the work of any previous chapters, and the ideas mentioned in the preliminary work section at the beginning of the book.

1. Suppose the ratio of male students to female students in a school is 21 : 19. If there are 720 students in the school altogether how many female students are there in the school?

2. Find the value of x in each of the following.

   (a) $\frac{x}{3} = \frac{7}{5}$ (b) $\frac{x}{10} = \frac{3}{4}$ (c) $\frac{5}{x} = \frac{4}{7}$ (d) $\frac{2}{x} = \frac{5}{7}$

3. Find m if $5(m + 3) + 2(3 - 2m) = 36$.

4. The mean of ten numbers is 10·8. Seven of these numbers have a mean of 12 and the other three numbers are a, (a + d) and (a + 2d).
   Determine (a + d).

5. A number of adults, some male and some female, were asked ten questions about a particular issue. The number answered correctly by the adults are shown in the frequency distribution table below and, below that is the frequency table for the females in the group.

Frequency distribution for entire group.											
Number correct	0	1	2	3	4	5	6	7	8	9	10
Frequency	3	1	4	12	20	25	25	32	16	14	13

Frequency table for the females in the group.											
Number correct	0	1	2	3	4	5	6	7	8	9	10
Frequency	0	0	0	10	17	10	8	9	7	12	13

   (a) How many females were asked the ten questions?
   (b) How many males were asked the ten questions?
   (c) Determine the range of the female scores.
   (d) Determine the range of the male scores.
   (e) Naomi claims that:

   The range of the male scores is bigger than the range of the female scores and therefore the male scores are more spread out than the female scores.

   Comment on her claim.

6. I think of a number, multiply it by two, add one and then double the result. To this answer I add on half of the number I first thought of and end up with 83. Find the number I first thought of.

7. To the nearest \$50, how much needs to be invested into an account paying 6% simple interest for the account to be worth at least \$19 000 in 5 years?

8. A company invests \$80 000, some into an account paying 6·3% per annum simple interest and the remainder in an account paying 5·4% per annum simple interest. After 2 years the \$80 000 had grown to \$89 612.
How much went into each account?

9. A maths exam was sat by 2145 candidates. The exam was marked out of 120 and the marks gained were distributed as follows.

Class interval	1 - 20	21 - 40	41 - 60	61 - 80	81 - 100	101 - 120
No. of candidates	113	263	340	720	478	231

(a) What is the midpoint of the 61– 80 class?
(b) Use the midpoint of each interval to determine a mean and standard deviation for this grouped distribution.

10. The scores achieved by two classes in a maths test are given below:

Class One

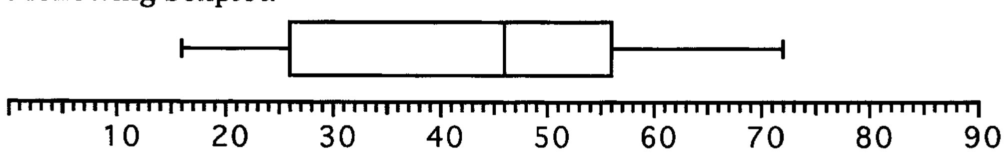
39	33	35	44	5	37	40	24	41	30	42
12	46	52	16	58	35	22	44	37	26	28
40	50	31								

Class Two

41	45	48	40	24	47	42	37	44	43	39
45	49	41	51	50	43	48	45	32	36	46

Draw a box and whisker diagram for each set of results and write a brief report comparing the distributions.

11. The scores obtained by the fifty students who sat a mathematics test gave rise to the following boxplot:



Construct a possible histogram for this set of fifty scores using the class intervals:
$10 \le$ score < 20, $20 \le$ score < 30, $30 \le$ score < 40, etc.

12. Two loads, load A and load B, of a particular valuable metal are for sale.
The weight of load A is greater than that of load B but load B is of a higher quality and therefore has a greater price per kilogram.
The ratio of the weights of the loads is as follows:

weight of load A : weight of load B = 15 : 8

The ratio of the per kilogram price of the two loads is as follows:

price of each kg of load A : price of each kg of load B = 3 : 5

If the price of load A is \$72 000 and the price of load B is \$$x$, find x.